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Numerical Analysis of Dynamical Contact Forces Due to Interaction of Moving Vehicles and Flexible Structures

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Abstract

In this study, the dynamic forces between the vehicle moving at a constant speed and the bridge considered as flexible structure have been analyzed in terms of many parameters affecting these forces (such as structure rigidity, vehicle velocity, vehicle body mass). The vehicle model used in the study was modeled as a six-degree-of-freedom model with passenger and driver seats. The bridge beam considered as a flexible structure in the study was modeled according to the Euler-Bernoulli beam theory with simple supported boundary conditions. The equation of motion of the system is obtained by Lagrange equation after the energy equations of the vehicle and bridge interaction are obtained. The differential equations are solved by the fourth order Runge-Kutta algorithm time in the time domain. Then dynamic forces originating from vehicle bridge interaction are analyzed in terms of many parameters. In conclusion, it has been revealed that the dynamic forces formed between the vehicle and the bridge are significantly influenced by the vehicle speed, body weight and bridge flexibility.

Key words: Lagrange equations, dynamic contact forces, Runge-Kutta algorithm, Euler-Bernoulli, vehicle bridge interaction.

1. Introduction

Dynamic behaviour of the structures under the influence of moving loads has been widely discussed in the literature as an important topic of interest. The studies [1–3] in which analytical solutions of various moving load problems are given are significant works on this subject. Neglecting the damping effects for a mass moving with a constant velocity on a simply supported beam, [4,5] investigated the subject and proposed some analytical solution methods. Considering the effect of the mass, [6–9] investigated dynamic behaviour of the different beams under accelerating mass influence. Due to the complexity of the modelling, analytic solutions to the moving mass problems have stayed insufficient. Instead, studies using the finite element model (FEM) approach proposed by [10–12] can be more useful in order to get more accurate solution results without neglecting the inertia and damping effects by modelling the moving mass as a time-dependent moving finite element in the finite element model of the entire system. Moving mass and structure interaction is also an important study subject in military applications of the mechanics, and in some studies [13,14] one can found some FEM and heuristic methods for determining muzzle displacements resulting from projectile and gun barrel interaction by considering the Coriolis, centripetal and inertia effects of the high-speed moving projectile inside

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the gun barrel. One of the essential field of application of moving load problems is the vehicle bridge interaction (VBI) problem, and in this respect [15–19] have presented a multi-axle vehicle bridge interaction model considering bridge dynamics and neglecting the effect of the interaction on vehicle components. For railroad design and other high speed infrastructure construction the effect of the moving mass is also another application field, thus [20,21] investigated the subject in terms of bridge dynamics. There are a number of studies on train-truck interaction problem by using the finite element adaptation, and the studies [18,22–25] have suggested some solutions of train-truck interaction using FEM. For predicting vehicle induced local responses of a skewed girder bridge [26], and for impact coefficient of mid-span continuous beam bridges interaction due to passage of heavy duty trucks and trains over flexible structures such as bridges [27] have investigated the interaction of heavy-vehicles with structures. Using a half car model, [28] has studied the passenger comfort considering a vehicle moving on a flexible structure with constant velocities

2. Mathematical modelling

To examine the effects of oscillations on vehicle dynamics, due to vehicle-bridge interaction, schematic half-car models with four and six degrees of freedom, as shown in Figures. (1a) and (1b), and the bridge system on which they were placed, were analysed.
The half-car model with six degrees of freedom contains front and rear tires with masses $m_{t1,2}$, driver and passenger seats with masses $m_{d,p}$, vehicle body with mass m_S and mass moment of inertia J, and connecting elements that consist of springs and dampers with the same features and linear characteristics. In the model shown in Figure 1a, k_{t1}, k_{t2}, k_{s1}, k_{s2}, k_d, and k_p represent, respectively, the stiffness of the front tire, rear tire, front suspension spring, rear suspension spring, driver seat spring, and passenger seat spring. Symbols c_{t1}, c_{t2}, c_{s1}, c_{s2}, c_d, and c_p, on the other hand, represent, respectively, the damping coefficients of the front tire, rear tire, front suspension, rear suspension, driver seat, and passenger seat. In addition, the mass of the suspension system is included in the mass of the vehicle body. The parameters y_{c1} and y_{c2} represent vertical movement of the tires at their point of contact with the bridge. These movements are affected by the deflection of the tires at their point of contact, and the roughness of the road at the same points. The vertical movement of the bridge $w_b(x, t)$ represents the deflection of any point x on the beam of the bridge at time t, relative to a reference point on the left-hand support of the beam. The symbol v represents the constant velocity of the vehicle as it moves from the left end of the beam to the right end. The function $r(x)$ represents the roughness of the bridge surface, and will be explained in detail in the following sections.

In the formulation for the VBI analysis following assumptions will be adopted:

- The bridge is modelled as a simple supported beam based on Euler-Bernoulli theory.
- The vehicle was modelled four and six DOF as lumped parameter.
- Only one car was accepted moving on the bridge with constant velocity v.
- The vehicle wheels are always in contact with the bridge surface when the vehicle is crossing over the bridge, and it is assumed that there is no separation when it passes through a bump or pit originating from any road failure.

- The road roughness function *r(x)* given by displacement function on the contact point between road and tire.

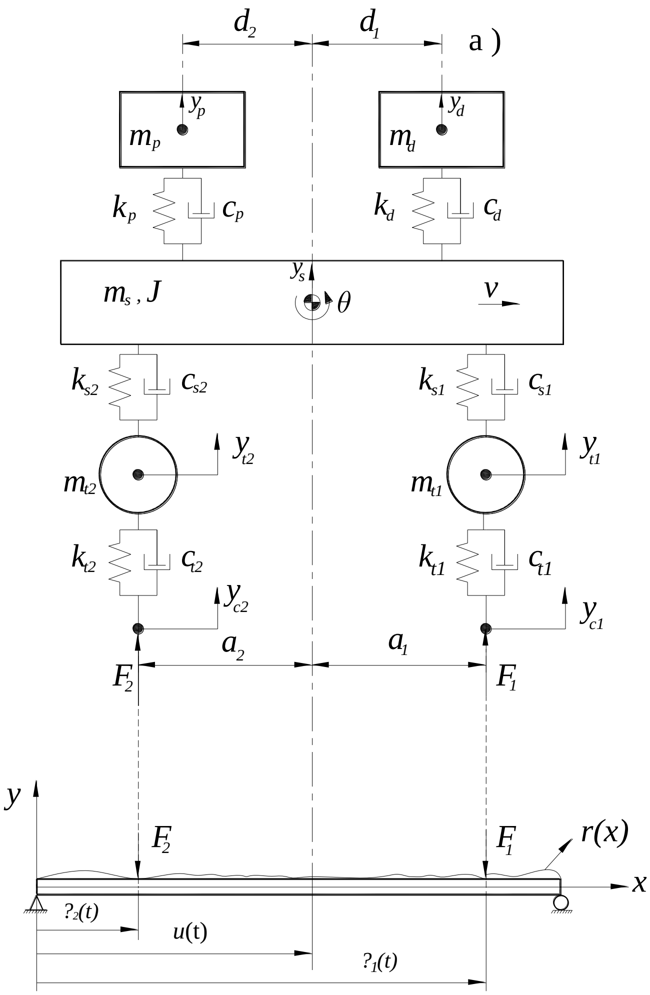

Figure 1. Model of a vehicle bridge interaction system (a) six DOF vehicle; (b) four DOF vehicle.

2.1. Deriving of equation of motion VBI system

With these assumptions, the kinetic and potential energy of the vehicle-bridge-passenger interaction, as shown in Figure 1a, is expressed as follows respectively:

$$E_k = \frac{1}{2}\left\{ \int_0^L \mu\left[\dot{w}_b^{\,2}(x,t) \right] dx + m_s \dot{y}_s^2(t) + J\dot{\theta}^2(t) + m_d \dot{y}_d^2(t) + m_p \dot{y}_p^2(t) + m_{t1}\dot{y}_{t1}^2(t) + m_{t2}\dot{y}_{t2}^2(t) \right\}, \qquad (1a)$$

$$E_p = \frac{1}{2}\left\{ \int_0^L EI[w_b''^2(x,t)dx] \; + k_d[y_s(t) + d_1\theta(t) - y_d(t)]^2 + k_p[y_s(t) - d_2\theta(t) - y_p(t)]^2 \right.$$
$$+ k_{s1}[y_s(t) + a_1\theta(t) - y_{t1}(t)]^2 + k_{s2}[y_s(t) - a_2\theta(t) - y_{t2}(t)]^2 + \tag{1b}$$
$$\left. + k_{t1}[y_{t1}(t) - y_{c1}]^2 H(x,\xi_1(t)) + k_{t2}[y_{t2}(t) - y_{c2}]^2 H(x,\xi_2(t)) \right\},$$

Here μ represents the mass per unit length of the bridge beam. In Eq. (1b), *EI* represents the rigidity of the bridge beam, and *H(x)* represents Heaviside shape functions. Front and rear tires' points of contact with the bridge beam are expressed as follows:

$$\xi_1(t) = u(t) + a_1, \qquad \xi_2(t) = u(t) - a_2, \tag{2}$$

Approaches, such as the principle of virtual work, Hamilton's principle, and D'Alembert's principle can be used for the equation of motion of the system, as shown in Figure 1a. This study uses Lagrange's equations, formed using the kinetic energy and potential energy equations of the vehicle-bridge integrated system, and the mode expansion method. The Galerkin equation for the deflection $w_b(x,t)$ of any point x on the beam at time t is expressed as follows:

$$w_b(x,t) = \sum_{i=1}^{n} \varphi_i(x)\eta_i(t),$$
$$\dot{w}_b(x,t) = \sum_{i=1}^{n} \varphi_i(x)\dot{\eta}_i(t), \tag{3a}$$
$$w_b''(x,t) = \sum_{i=1}^{n} \varphi_i''(x)\eta_i(t),$$

$$\varphi_i(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{i\pi x}{L}\right), \quad i = 1,2,...,n. \tag{3b}$$

Here, η_i is the generalized coordinate representing the deflection of the beam element, and φ_i is the shape of oscillation that is obtained with the boundary conditions of the bridge beam. Orthogonality conditions between these shapes of oscillation are expressed, as in Eq. (5).

$$\int_0^L \mu\varphi_i(x)\varphi_j(x)dx = N_i\delta_{ij}, \quad \int_0^L EI\varphi_i''(x)\varphi_j''(x)dx = \Pi_i\delta_{ij}, \tag{4}$$

In Eq. (4), δ_{ij} represents the Kronecker delta where (i,j=1,2,…,n) and and N_i and Π_i are defined by setting $i=j$ in Eq. (5).. The axle load of the vehicle during contact with the bridge is expressed with Heaviside functions, over time, as follows:

$$f_c(x,t) = -(f_{c1}H(x - \xi_1(t))) + (f_{c2}H(x - \xi_2(t))), \tag{1a}$$

$$f_{c1} = \left(m_{t1} + m_s \frac{a_2}{a_1 + a_2} + m_d \frac{a_2 + d_1}{a_1 + a_2} + m_p \frac{a_2 - d_2}{a_1 + a_2} \right) g, \tag{5a}$$

$$f_{c2} = \left(m_{t2} + m_s \frac{a_1}{a_1 + a_2} + m_d \frac{a_1 - d_1}{a_1 + a_2} + m_p \frac{a_1 + d_2}{a_1 + a_2} \right) g, \tag{5b}$$

Rayleigh dissipation function for the vehicle-bridge integrated system is expressed as follows:

$$\begin{aligned} D = \frac{1}{2} \Bigg\{ & \int_0^L c \dot{w}_b^{\,2}(x,t) dx + c_{t1} [\dot{y}_{t1}(t) - \dot{w}_b(\xi_1(t),t)]^2 H(x - \xi_1(t)) \\ & + c_p [\dot{y}_s(t) - d_2 \dot{\theta}(t) - \dot{y}_p(t)]^2 + c_{s1} [\dot{y}_s(t) + a_1 \dot{\theta}(t) - \dot{y}_{t1}(t)]^2 \\ & + c_{s2} [\dot{y}_s(t) - a_2 \dot{\theta}(t) - \dot{y}_{t2}(t)]^2 + c_d [\dot{y}_s(t) + d_1 \dot{\theta}(t) - \dot{y}_d(t)]^2 + c_{t2} [\dot{y}_{t2}(t) - \dot{w}_b(\xi_2(t),t)]^2 H(x - \xi_2(t)) \Bigg\}, \end{aligned} \tag{6}$$

In Eq. (6), c is the equivalent damping function for bridge beam. In addition, the Lagrange equation ($L=E_k-E_p$) of the system is equal to the difference between the kinetic energy and the potential energy. If the Lagrange equation is rearranged for six independent coordinates, the following is obtained:

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{p}_k(t)} \right) - \frac{\partial L}{\partial p_k(t)} + \frac{\partial D}{\partial \dot{p}_k(t)} = 0 \;, \; k = 1, 2, ..., 6, \tag{7a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\partial L}{\partial \dot{\eta}_i(t)} \right) - \frac{\partial L}{\partial \eta_i(t)} + \frac{\partial D}{\partial \dot{\eta}_i(t)} = Q_i \;, \quad \mathrm{i} = 1, 2, 3, 4, \tag{7b}$$

2.2. The forces act on vehicle due to vehicle bridge coupled vibrations

Define abbreviations that are not standard in this field in a footnote to be placed on the first page of the article [3-4]. Such abbreviations that are unavoidable in the abstract must be defined at their first mention there, as well as in the footnote. Ensure consistency of abbreviations throughout the article [3, 8].

The static forces that are put on the points of contact between the bridge and the front and rear axles of the vehicle are expressed as in Eqs. (5a-b). In this case, the forces at the points of contact of the tires are expressed as follows:

$$F(x,t)_i = f_{ci} - k_{ti}(y_{ti} - y_{ci}) - c_{ti}(\dot{y}_{ti} - \dot{y}_{ci}), \quad i = 1, 2, \tag{8}$$

If the goal is to analyse the forces acting upon the front and rear axles, the static load Eqs. (5a-b) developed for the points of contact of the front and rear tires of the vehicle needs to be re-arranged. In this case, front and rear axle loads are expressed as in Eqs. (9a-b), by subtracting the masses of the axles.

$$F(x,t)_i = f_{ci} - k_{ti}(y_{ti} - y_{ci}) - c_{ti}(\dot{y}_{ti} - \dot{y}_{ci}), \quad i = 1, 2, \tag{9a}$$

$$F(x,t)_i = f_{ci} - k_{ti}(y_{ti} - y_{ci}) - c_{ti}(\dot{y}_{ti} - \dot{y}_{ci}), \quad i = 1, 2, \tag{9b}$$

3. Results

To test the accuracy of the theory presented, the half-car model with four degrees of freedom, shown in Figure 1b, and the bridge and vehicle parameters given in Table 1 were used. Figures (3a-c) provides a comparison of graphs for vehicle body's displacement (y_s), speed, and acceleration with reference [29] using the parameters given in Table 1. An examination of these graphs shows that the results overlap significantly with the reference work. In these analyses, the dynamic effects of the road roughness function on the vehicle, which is an indicator of the quality of asphalt on the bridge, were not taken into consideration.

Table 1. The numerical values of the VBI used in this study.

Vehicle parameters			
m_s (kg)	1794.4	k_{t2} (N/m)	101115
m_{t1} (kg)	87.15	k_d (N/m)	14000
m_{t2} (kg)	140.4	d_1 (m)	0.481
m_d (kg)	75	k_p (N/m)	14000
m_p (kg)	75	c_{s1} (Ns/m)	1190
J (kgm^2)	3443.04	c_{s2} (Ns/m)	1000
a_1 (m)	1.271	c_{t1} (Ns/m)	14.6
d_2 (m)	1.313	c_{t2} (Ns/m)	14.6
k_{s1} (N/m)	66824.2	c_d (Ns/m)	50.2
k_{s2} (N/m)	18615	c_p (Ns/m)	62.1
k_{t1} (N/m)	101115	a_2 (m)	1.713
Bridge parameters			
L (m)	100	E(GPa)	207
I (m^4)	0.174	μ (kg/m)	20000
C (Ns/m)	1750		

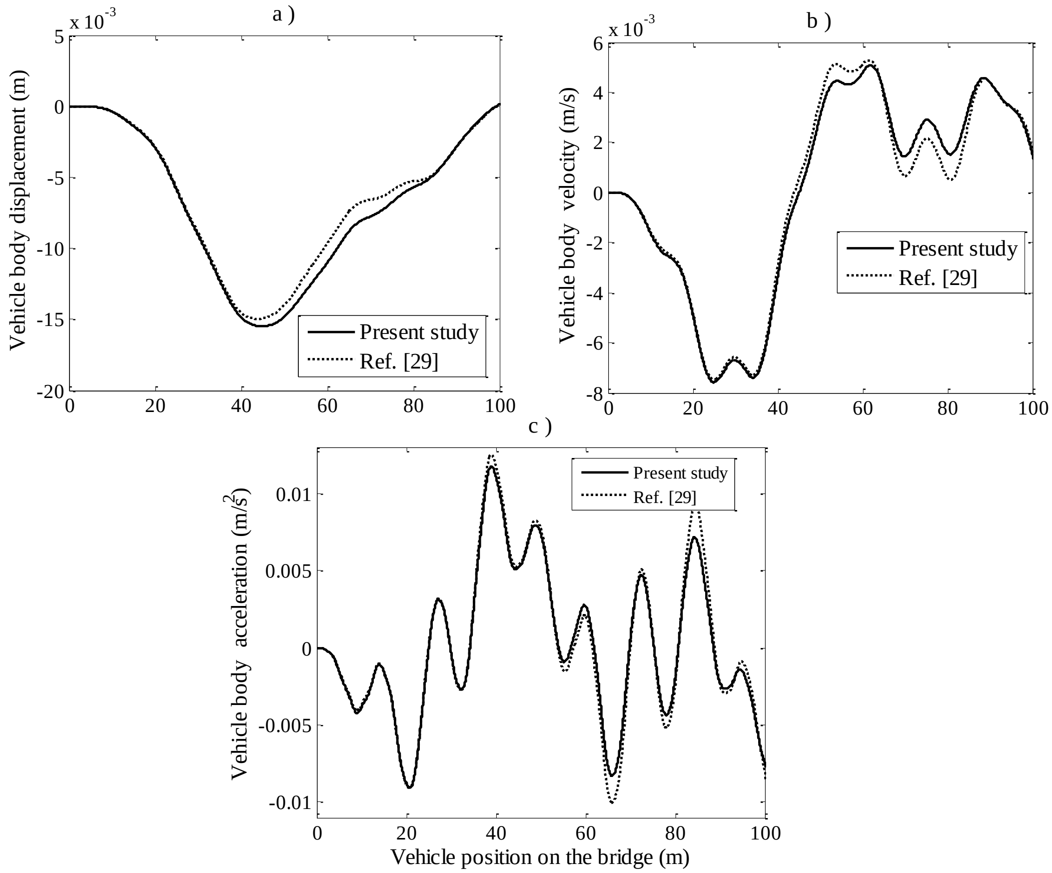

Figure 3. The vehicle body (a) displacement (m); (b) velocity (m/s); (c) acceleration (m/s^2) for time step size Δt=0.001 s, vehicle velocity v=40 km/h.

In this section, forces acting on the vehicle due to bridge-vehicle coupled vibrations, as the vehicle continues its motion over the bridge, are analysed. Taking both the static and the dynamic loads of the vehicle into account, forces acting on the points of contact of the front and rear tires, front and rear axles, vehicle body, and driver and passenger seats were analysed for different vehicle speeds and road roughness values. As a vehicle moves over a bridge, the vehicle on the bridge applies two types of force. One is the static load that the masses of passenger and driver seats, vehicle body, and front and rear axles apply on the tires. The other is the dynamic force resulting from the changes in shape, and from the deflection that are due to the flexibility of the tires, when the vehicle is in motion. These two forces act in opposite directions to one another at the point of contact of the tires. The equations of contact forces between wheels and pavement, and other dynamical forces acted on vehicle components such as front and rear axles, car body, driver and passenger seats are given in reference [30].

In Figures 4a-d, the vehicle passage speed over the bridge is increased from 2 m/s to 70 m/s with 2 m/s increments, and show the front-rear wheel contact point, front-rear axles, maximum forces variation of passenger and driver seats. This analysis is carried out by neglecting the roughness on the bridge ($G_d\,(n_0)$=0) and at the time step of Δt=0.001 s, and with considered mode number *n*=4. As Figures. (4a-d) show, increase of vehicle speed passing on the bridge caused an increase in dynamic forces impacting the vehicle.

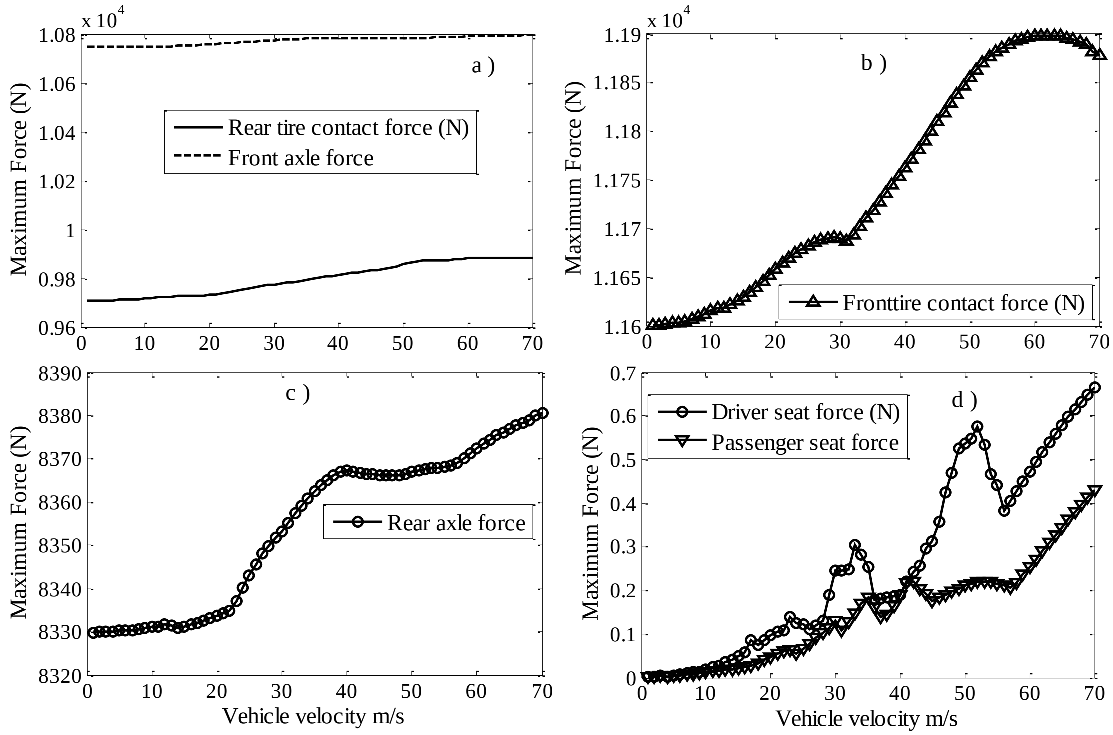

Figure 4. The effect of vehicle velocity upon dynamic forces act on vehicle for time step size Δt=0.001 s (a) Rear tire contact and front axle forces; (b) Front tire contact force (c) rear axle force; (d) driver and passenger seats forces.

Conclusions

In this study, a precise model of vehicle structure interaction has been presented and some results of various parameters affecting the interaction have been presented. The dynamic forces on the vehicle components caused by vehicle bridge interaction during the passage of vehicle has analysed individually for different velocities and roughness (G_d (n_0)) coefficients and the results have been presented. The proposed method, taking into account all road conditions including bridges, random or definite surface roughness and a vehicle configuration including front and rear axles, tire stiffness, suspension system, car body, driver and passenger seats, is a full scale analysis tool and can be very useful for both vehicle design engineers and structural engineers. For a given road scenario, for example 100.000 km driving of the vehicle, one can easily model the scenario and convert the results of dynamic forces into a time dependent or a spatial dependent force function for the usage for an acceptance tests of vehicle components.

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